

Abelian varieties & Galois actions

July 25 - 27, 2017

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ABSTRACTS

Speaker : Sara Arias-de-Reyna

Title : *Ordinary abelian varieties and the Inverse Galois Problem*

Abstract : Given an n -dimensional abelian variety A/\mathbb{Q} which is principally polarised, we consider for each prime number ℓ the representation of the absolute Galois group of the rational numbers, $\rho_{A,\ell} : G_{\mathbb{Q}} \rightarrow \mathrm{GSp}(2n, \ell)$ attached to the ℓ -torsion points of A . Provided the representation is surjective, we obtain a realisation of $\mathrm{GSp}(2n, \ell)$ as the Galois group of the finite extension $\mathbb{Q}(A[\ell])/\mathbb{Q}$, and the ramification type of a prime p in this extension can be read off from the type of reduction of A at p . In this talk we address the question of producing tame Galois realisations of $\mathrm{GSp}(2n, \ell)$ by making use of those representations, and determine a series of local conditions ensuring tameness and surjectivity. In particular, we will work with abelian varieties ordinary at ℓ . However, it is not clear how to set up the local conditions to force the existence of a global abelian variety (defined over \mathbb{Q}) satisfying all of them simultaneously. In the cases when $n \leq 3$, we can make use of Jacobians of curves in a family, and deform the curves p -adically modulo a finite set of primes p to guarantee the local conditions hold, thus obtaining tame Galois realisations of $\mathrm{GL}_2(\mathbb{F}_{\ell})$, $\mathrm{GSp}_4(\mathbb{F}_{\ell})$ and $\mathrm{GSp}_6(\mathbb{F}_{\ell})$. For higher values of n , new ideas are required.