Abelian varieties & Galois actions

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ABSTRACTS

Speaker : Sebastian Petersen
Title : *A variational open image theorem in positive characteristic*
Abstract : Let k be a finitely generated infinite field of characteristic $p > 0$, S a smooth geometrically connected k-variety of positive dimension and $f : \mathscr{X} \to S$ a smooth proper morphism of schemes. Let $K = k(S)$ be the function field of S and let $X/k(S)$ be the generic fibre of $\mathscr{X}$. Fix $j \in \mathbb{N}$. For every prime number $\ell \neq p$ we define $V_\ell := H^j(X_{\overline{K}}, \mathbb{Q}_\ell)$ and let

$$\rho_{\ell^\infty} : \pi_1(S) \to \underline{\mathrm{GL}}_{V_\ell}(\mathbb{Q}_\ell)$$

be the representation of $\pi_1(S)$ on the $\mathbb{Q}_\ell$-vector space V_ℓ. We write

$$\rho : \pi_1(S) \to \prod_{\ell \neq p} \underline{\mathrm{GL}}_{V_\ell}(\mathbb{Q}_\ell)$$

for the induced adelic representation $\prod_{\ell \neq p} \rho_{\ell^\infty}$. For every point $s \in S$ with residue field $k(s)$ we denote by $s_* : \mathrm{Gal}(k(s)) \to \pi_1(S)$ the homomorphism induced by s (well-defined up to conjugation), and for any group homomorphism $\tau : \pi_1(S) \to H$, we define $\tau_s := \tau \circ s_*$. Finally, for every prime number $\ell \neq p$ let $\underline{G}(\rho_{\ell^\infty,s})$ (resp. $\underline{G}(\rho_{\ell^\infty})$) be the connected component of the Zariski closure of $\rho_{\ell^\infty,s}(\mathrm{Gal}(k(s)))$ (resp. of $\rho_{\ell^\infty}(\pi_1(S))$) in $\underline{\mathrm{GL}}_{V_\ell}/\mathbb{Q}_\ell$, and define $S^{\mathrm{gen}}(\rho_{\ell^\infty}) = \{s \in S \text{ a closed point} : \underline{G}(\rho_{\ell^\infty,s}) = \underline{G}(\rho_{\ell^\infty})\}$.

Theorem.

(a) The sets $S^{\mathrm{gen}}(\rho_{\ell^\infty})$ are independent of ℓ. Let $S^{\mathrm{gen}}(\mathscr{X}/S) := S^{\mathrm{gen}}(\rho_{\ell^\infty})$ for any $\ell \neq p$.

(b) The set $S^{\mathrm{gen}}(\mathscr{X}/S)$ is Zariski dense in S, and in particular it is infinite.

(c) The group $\rho_s(\mathrm{Gal}(k(s)))$ is open in $\rho(\pi_1(S))$ for every $s \in S^{\mathrm{gen}}(\mathscr{X}/S)$.

This Theorem parallels the variational open image theorem of Cadoret for abelian schemes over varieties over finitely generated fields of characteristic zero published in 2016. In positive characteristic there is a result similar to our main theorem in a very recent preprint of Cadoret. (This is a joint work with Gebhard Böckle and Wojciech Gajda.)