Abelian varieties & Galois actions

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Collegium Minus
Adam Mickiewicz University
Poznań, Poland

ABSTRACTS

Speaker : Yuri Zarhin
Title : *Endomorphism algebras of abelian varieties with reference to superelliptic jacobians.*
Abstract : I'll explain how one may obtain an information about the structure of endomorphism algebras of abelian varieties, using ideas from group theory and Galois theory. In particular, there will be a plenty of explicit examples of abelian varieties with *small* endomorphism ring.

Speaker : Emmanuel Ullmo
Title : *Algebraic flows on Abelian varieties*
Abstract : Let A be a complex Abelian variety. The Abelian Ax-Lindemann theorem says that the Zariski closure of an algebraic flow on A is a translate of an Abelian subvariety. I'll discuss results concerning the closure of such an algebraic flow in the ordinary topology.

Speaker : Tim Dokchitser
Title : *Models of curves*
Abstract: I will explain an approach to constructing regular and semistable models of curves over DVRs. It is based on the ideas that are well-known for varieties over the complex numbers in toric and tropical geometry, but can be imported to curves over arithmetic schemes as well. This often allows to compute important invariants of the curve – conductor, semistable type, etale cohomology etc. in an elementary way.

Speaker : Jeff Achter
Title : *Distinguished models of intermediate Jacobians*
Abstract: Consider a smooth projective variety over a number field. The image of the associated (complex) Abel-Jacobi map inside the (transcendental) intermediate Jacobian is a complex abelian variety. We show that this abelian variety admits a distinguished model over the original number field, and use it to address a problem of Mazur on modeling the cohomology of an arbitrary smooth projective variety by that of an abelian variety. We also recover an old theorem of Deligne on intermediate Jacobians of complete intersection varieties, and describe new invariants of derived equivalent threefolds. (This is joint work with Sebastian Casalaina-Martin and Charles Vial.)

Speaker : Sebastian Petersen
Title : *A variational open image theorem in positive characteristic*
Abstract : Let k be a finitely generated infinite field of characteristic $p > 0$, S a smooth geometrically connected k-variety of positive dimension and $f : \mathscr{X} \to S$ a smooth proper morphism of schemes. Let $K = k(S)$ be the function field of S and let $X/k(S)$ be the generic fibre of $\mathscr{X}$. Fix $j \in \mathbb{N}$. For every prime number $\ell \neq p$ we define $V_\ell := H^j(X_{\overline{K}}, \mathbb{Q}_\ell)$ and let

$$\rho_{\ell^\infty} : \pi_1(S) \to \underline{\mathrm{GL}}_{V_\ell}(\mathbb{Q}_\ell)$$

be the representation of $\pi_1(S)$ on the $\mathbb{Q}_\ell$-vector space V_ℓ. We write

$$\rho : \pi_1(S) \to \prod_{\ell \neq p} \underline{\mathrm{GL}}_{V_\ell}(\mathbb{Q}_\ell)$$

for the induced adelic representation $\prod_{\ell \neq p} \rho_{\ell^\infty}$. For every point $s \in S$ with residue field $k(s)$ we denote by $s_* : \mathrm{Gal}(k(s)) \to \pi_1(S)$ the homomorphism induced by s (well-defined up to conjugation), and for any group homomorphism $\tau : \pi_1(S) \to H$, we define $\tau_s := \tau \circ s_*$. Finally, for every prime number $\ell \neq p$ let $\underline{G}(\rho_{\ell^\infty,s})$ (resp. $\underline{G}(\rho_{\ell^\infty})$) be the connected component of the Zariski closure of $\rho_{\ell^\infty,s}(\mathrm{Gal}(k(s)))$ (resp. of $\rho_{\ell^\infty}(\pi_1(S))$) in $\underline{\mathrm{GL}}_{V_\ell}/\mathbb{Q}_\ell$, and define $S^{\mathrm{gen}}(\rho_{\ell^\infty}) = \{s \in S \text{ a closed point} : \underline{G}(\rho_{\ell^\infty,s}) = \underline{G}(\rho_{\ell^\infty})\}$.

Theorem.

(a) The sets $S^{\mathrm{gen}}(\rho_{\ell^\infty})$ are independent of ℓ. Let $S^{\mathrm{gen}}(\mathscr{X}/S) := S^{\mathrm{gen}}(\rho_{\ell^\infty})$ for any $\ell \neq p$.

(b) The set $S^{\mathrm{gen}}(\mathscr{X}/S)$ is Zariski dense in S, and in particular it is infinite.

(c) The group $\rho_s(\mathrm{Gal}(k(s)))$ is open in $\rho(\pi_1(S))$ for every $s \in S^{\mathrm{gen}}(\mathscr{X}/S)$.

This Theorem parallels the variational open image theorem of Cadoret for abelian schemes over varieties over finitely generated fields of characteristic zero published in 2016. In positive characteristic there is a result similar to our main theorem in a very recent preprint of Cadoret. (This is a joint work with Gebhard Böckle and Wojciech Gajda.)

Speaker : Chun Yin Hui
Title : *The abelian part of a compatible system*
Abstract : Let K be a number field and $\{V_\ell\}$ a strictly compatible system of semisimple Galois representations of K arising from geometry. Denote by V_ℓ^{ab} and G_ℓ respectively the maximal abelian subrepresentation and algebraic monodromy group of V_ℓ. We give conditions on G_ℓ such that the abelian subsystem $\{V_\ell^{ab}\}$ is also strictly compatible, which is conjectured to hold.

Speaker : Chris Hall
Title : *Soft Hilbert Irreducibility Theorems*
Abstract : Let G be the Galois group of a finite Galois extension L/K of the rational function field $K = \mathbb{Q}(t)$. The extension L/K gives rise to a one-parameter family of subgroups $G_s \subset G$ indexed by rational values s in $\mathbb{Q}$. It is well understood that the G_s is well-defined up to conjugacy in G and that the set of s in $\mathbb{Q}$ such that $G_s \subset G$ is a thin set. We will discuss a soft technique for finding explicit s in $\mathbb{Q}$ satisfying $G_s = G$. We will also explain how to apply these techniques to extensions coming from arithmetic geometry.

Speaker : Stefan Patrikis
Title : *Compatible families of Galois representations with E6 monodromy*
Abstract : I will explain the construction of the first compatible families of Galois representations having algebraic monodromy group equal to the exceptional group E_6. The technique combines deformations of Galois representations with potential automorphy theorems. The resulting compatible systems moreover appear in the cohomology of (Shimura) varieties. (This is joint work with G. Boxer, F. Calegari, M. Emerton, B. Levin, and K. Madapusi Pera.)

Speaker : Sara Arias-de-Reyna
Title : *Ordinary abelian varieties and the Inverse Galois Problem*

Abstract : Given an n-dimensional abelian variety $A/\mathbb{Q}$ which is principally polarised, we consider for each prime number ℓ the representation of the absolute Galois group of the rational numbers, $\rho_{A,\ell} : G_{\mathbb{Q}} \to \mathrm{GSp}(2n, \ell)$ attached to the ℓ-torsion points of A. Provided the representation is surjective, we obtain a realisation of $\mathrm{GSp}(2n, \ell)$ as the Galois group of the finite extension $\mathbb{Q}(A[l])/\mathbb{Q}$, and the ramification type of a prime p in this extension can be read off from the type of reduction of A at p. In this talk we address the question of producing tame Galois realisations of $\mathrm{GSp}(2n, \ell)$ by making use of those representations, and determine a series of local conditions ensuring tameness and surjectivity. In particular, we will work with abelian varieties ordinary at ℓ. However, it is not clear how to set up the local conditions to force the existence of a global abelian variety (defined over $\mathbb{Q}$) satisfying all of them simultaneously. In the cases when $n \leq 3$, we can make use of Jacobians of curves in a family, and deform the curves p-adically modulo a finite set of primes p to guarantee the local conditions hold, thus obtaining tame Galois realisations of $\mathrm{GL}_2(\mathbb{F}_\ell)$, $\mathrm{GSp}_4(\mathbb{F}_\ell)$ and $\mathrm{GSp}_6(\mathbb{F}_\ell)$. For higher values of n, new ideas are required.

Speaker : Johan Commelin
Title : *On compatibility of the ℓ-adic realisations of abelian motives*
Abstract : In the sixties, Serre introduced the concept of a compatible system of Galois representations. Since Deligne proved the Weil conjectures, we know that the l-adic etale cohomology groups of a smooth projective variety over a number field form such a compatible system. The analogous statement for the ℓ-adic realisations of a motive (in the sense of Andre, or absolute Hodge cycles) is not known in general. I will introduce the concept of quasi-compatibility, a slightly relaxed version on the original condition. Familiar notions, such as Frobenius tori, are still accessible under this weaker condition. I will show how a recent result of Kisin may be used to show that the l-adic realisations of an abelian motive (in the sense of Andre, or absolute Hodge cycles) give rise to an E-rational quasi-compatible system of Galois representations. If time permits, I will mention some applications of the main theorem to the Mumford-Tate conjecture.

Speaker : Davide Lombardo
Title : *Galois representations attached to abelian varieties: effective aspects*
Abstract: Let A be an abelian variety defined over a number field K. To A/K one can canonically attach a family (ρ_ℓ) of ℓ-adic Galois representations, which have long been known to carry significant arithmetic information about A. Under various combinations of hypotheses concerning the dimension and the endomorphism algebra of A, results of Serre, Pink, Ribet, and others show that – for every ℓ – the image G_ℓ of ρ_ℓ is open in $\mathrm{MT}(A)(\mathbb{Z}_\ell)$, where $\mathrm{MT}(A)$ is the Mumford-Tate group of A. This gives a description of G_ℓ "up to finite index", and in many cases one even knows that the equality $G_\ell = \mathrm{MT}(A)(\mathbb{Z}_\ell)$ holds for all sufficiently large primes ℓ. In this talk I will consider the problem of making such results *effective*, giving for example an explicit value $B(A/K)$ – expressed as a simple function of A and K – such that the equality $G_\ell = \mathrm{MT}(A)(\mathbb{Z}_\ell)$ holds for all $\ell > B(A/K)$.

Speaker : David Zywina
Title : *A radical characterization of abelian varieties*
Abstract : A famous theorem of Faltings says that an abelian variety A over a number field K is determined, up to isogeny, by its Frobenius polynomials $P_{A,\mathfrak{p}}(x)$ for almost all primes $\mathfrak{p}$. We will discuss analogous results with weaker conditions. For example, an abelian variety A/K is determined, up to isogeny, by the cardinality $|A(\mathbb{F}_\mathfrak{p})| = P_{A,\mathfrak{p}}(1)$ for almost all $\mathfrak{p}$. We will also discuss recent work of Theodore Hui which shows that the simple quotients of A, over an explicit extension L of K, are determined by the radical of $|A(\mathbb{F}_\mathfrak{p})|$ for almost all $\mathfrak{p}$; this extends results of Hall, Perucca and Ratazzi.